Upscaling of Macroscopic Properties from the Pore to Continuum Scale using Mortars

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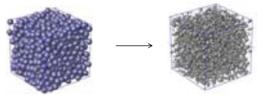
Abstract

Pore-scale network modeling has become an effective method for accurate prediction and upscaling of macroscopic properties (such as permeability, relative permeability, and capillary pressure) in porous media. In many cases these models compare favorably to experimental measurements. However, computational and imaging restrictions generally limit the network size to the order of 1.0 mm^3 (few thousand pores). For extremely heterogeneous media these models are not large enough to capture the petrophysical properties of the entire medium and inaccurate results can be obtained when upscaling to the continuum scale. Moreover, the boundary conditions imposed are artificial; a pressure gradient is imposed in one dimension so the influence of flow behavior in the surrounding media is not included.

In this work we upscale permeability for single-phase flow in heterogeneous media using pore-scale network models. A more efficient, novel domain decomposition method is used for upscaling the permeability of pore-scale models. The medium is decomposed into hundreds of smaller networks (sub-domains) and then coupled with the surrounding models to determine accurate boundary conditions. Finite element mortars are used as a mathematical tool to ensure interfacial pressures and fluxes are matched at the interfaces of the networks boundaries. The results compare favorably to the more computationally intensive (and impractical) approach of upscaling the media as a single model. Moreover, the results are much more accurate than traditional hierarchal upscaling methods. This upscaling technique has important implications for using pore-scale models directly in reservoir simulators in a multiscale setting. Moreover, we develop some a priori upscaling techniques (super permeability tensor) that allow for fast coupling between the models in the multiscale simulator.

Pore-Scale Models as Stand-Alone Tools

Pore-scale network models can be obtained directly from real media or from computer generated models. In the last few decades they have become more representative of real media and more detailed transport physics are included



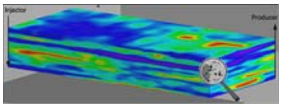
Predictive networks can be used to obtain macroscopic properties in porous media and thus the models can be used as surrogates for experimental tests

- Permeability
- Capillary Pressure
- Effective viscosity for non-Newtonian fluids
- Dispersion coefficients
- Non-Darcy Coefficient

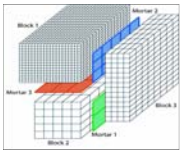
But…

- Is direct upscaling suffient?
- Shouldn't the boundary conditions depend on flow behavior?
- How can we include pore-scale models in a multiscale setting

Modeling Through the Magnifying Glass:



Mortar Coupling

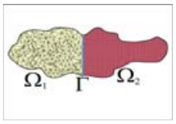


2D FEM mortar spaces are used at interfaces to iteratively match pressures at fluxes

Designed to allow for coupling of different models, physics, and scales

Improve accuracy by using finer meshes and higher-order mortars

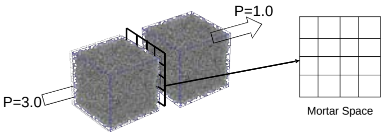
Subdomains are decomposed and solved separately using interface pressure bcs



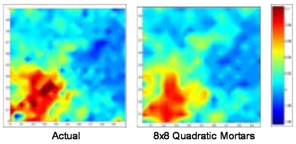
The subdomains can be different models or even scales, which would allow us to couple pore and continuum-scale models

Mortar Coupling of Single-Phase Flow

Until recently, mortars were used exclusively to couple domains in continuum simulators. Here, we couple pore-scale network models to other pore-scale or macro-scale models to ensure the correct boundary conditions are imposed

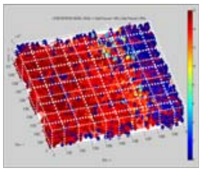


The FEM mortars iteratively determine boundary conditions in such a way that the pressure and flux fields match in a weak sense. The below figure shows that pressure field at the interface obtained using mortars is a good approximation to the true solution.


Actual
8x8 Quadratic Mortars

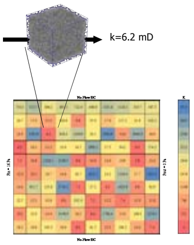
Mortar Upscaling of Permeability

The objective is to test the effect of surrounding media on macroscopic properties (permeability) and develop a method to upscale permeability accurately by coupling to adjacent media using mortars. The method is shown to be more accurate than a straightforward hierarchal upscaling approach in which permeability is upscaled on small, stand-alone network models and substituted into a finite difference model. The mortar approach is demonstrated through a comparison of three numerical experiments.

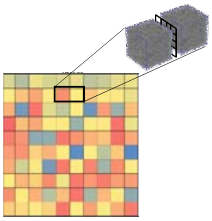


Numerical Test #1. Permeability is calculated in a large (~10^6 pores), heterogeneous and anisotropic network model. The results are assumed correct for the chosen boundary conditions and the upscaled permeability, K_{true}, is used as a reference.

Numerical Test #2. Network is divided into 100 sub-networks and each sub-network is treated as a stand-alone tool (with simple, artificial boundary conditions); sub-network permeabilities are extracted and placed on a finite difference grid. K_{FD} for the entire domain is then back-calculated using Darcy's law and the finite-difference solution for flow.



$$\nabla \cdot \left(\mathbf{K} \nabla P\right) = 0$$



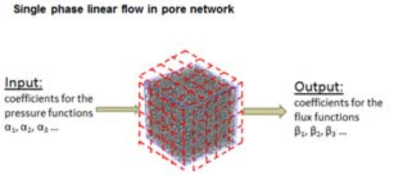
Numerical Test #3. Flow in each sub-network is computed individually, but boundary conditions are found through mortar coupling. The pressure field at shared boundaries is chosen so that fluxes match weakly. K_{mortar} is then calculated for the entire domain.

Table 1. Mortar Upscaling can give more accurate upscaled results

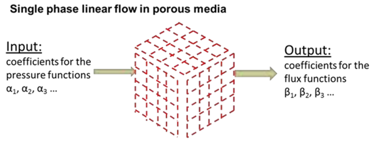
		K_{true}	K_{FD}	$K_{\text{mortar-constant}}$				$K_{\text{mortar-linear}}$			
				1x1	2x2	3x3	4x4	1x1	2x2	3x3	4x4
Network A	Kx	37.54	32.39	65.52	49.27	43.44	40.35	40.01	39.05	38.09	36.9
	Ky	44.69	36.21	80.23	59.94	53.34	49.2	48.62	47.29	46	44.55
Network B	Kx	101.04	81.08	159.58	128.46	110.73	105.89	106.52	104.08	101.04	97.4
	Ky	37.1	28.99	54.24	42.83	37.93	37.04	37.06	36.48	35.7	34.79

Upscaling Pore-Scale Models—Super Permeability Matrix

Pore networks are coupled together using mortars. Each network is solved locally. The pore network is provided with the input pressure profile, and output the flux profile on six faces. Furthermore, since the pressure profile and flux profile are all described through basis functions on the mortar space, every pore network communicates with the coefficients of the basis function for pressure (a set of α) and only output the coefficients for the basis function regarding the flux (a set of β).

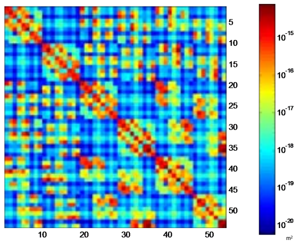

Single phase linear flow in pore network

If we consider the pore network simulation as a black box, then we actually have :


Single phase linear flow in porous media

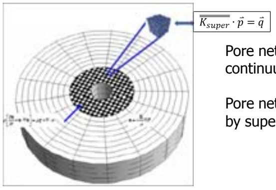
$$\overline{\overline{K_{super}}} \cdot \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

a. Construct pressure vector of α such as [0,0,…1,0,0,…..0]′ (i^{th} value of element is 1)
b. Input this vector to pore-network simulation
c. Compute flux value at the boundary pores
d. Project flux value back to the basis functions on mortar space and calculate β vector.
e. This β vector is the i^{th} row of the representative matrix: A(i,:)
f. Repeat step a to e for i=1,2,….N_{bs}



Features of the super permeability matrix:
- A 5000 pore network can be substituted by a 54x54 matrix
- diagonal elements are much greater than off-diagonal elements
- Value of each elements indicates the connectivity between two regions
- Element values range 5 orders of magnitude

Multiscale Simulator Near Well bore



$$\overline{\overline{K_{super}}} \cdot \vec{p} = \vec{q}$$

Pore networks are coupled with continuum and network model

Pore network simulation are substituted by super permeability tensor

Conclusions and Future Work

1. Pore-scale models are usually utilized as stand-alone tools, but artificial boundary conditions are employed and therefore incorrect macroscopic properties can be obtained via upscaling
2. Boundary conditions can be obtained through mortar coupling to surrounding media; upscaled permeability values are found to be more accurate
3. Expensive pore-scale simulation could be replaced by a super permeability tensor

Acknowledgment

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