Scale-Up of Flow/Transport Processes in Heterogeneous Media

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Abstract

Reservoir heterogeneities manifest themselves over a wide range of length scales, and their interaction with various transport mechanisms control the performance of subsurface flow and transport processes. Modeling these processes at large-scales requires proper scale-up of heterogeneity and its interaction with underlying transport mechanisms. This paper demonstrates a new technique to systematically quantify the scaling characteristics of transport processes in single-phase systems by accounting for sub-scale heterogeneities and their interaction with various transport mechanisms based on the volume averaging approach.

Introduction

There are two major approaches that can be employed to understand the scale-up characteristics of recovery responses. One way is to utilize dimensionless groups, which assess the relative magnitudes of different transport mechanisms corresponding to mean reservoir parameters. Recovery performance is assumed to scale according to these dimensionless groups. The major drawback with dimensionless group approach is that detailed information regarding spatial correlation of reservoir parameters is not incorporated; the variability exhibited by the fine-scale geologic model is lumped into a single dimensionless number.

The second approach is to evaluate the scale-up characteristics by deriving the full set of flow/mass transfer equations at the macroscopic scale as averages of fine-scale quantities. The two main classes of such techniques are 1) Ensemble Averaging Methods (Neuman et al., 1987; Zhang and Winter, 1999; Vereecken, 2007) and 2) Volume Averaging Methods (Whitaker, 1999). As opposed to the method of ensemble averaging, volume averaging methods deal with spatial averages of a random field. This method is useful for deriving statistics (mean or variance) of effective transport parameters (provided that we account for reservoir heterogeneity) as a function of length scale.

Project Objective

Develop a framework for relating heterogeneity & scaling characteristics of reservoir attributes + transport processes to uncertainty in flow performance prediction at a particular scale

Method

The governing equation and boundary and initial conditions can be written as (Quintard and Whitaker, 1994):

$$\frac{\partial C_{i\beta}}{\partial t}+\nabla\cdot\left(\mathbf{u}_\beta C_{i\beta}\right)=\nabla\cdot\left(D\nabla C_{i\beta}\right)$$

B.C. 1: $C_\beta = C_{eq}$ at $A_{\beta\gamma}$ (zero concentration gradient across the β-γ interface)

B.C. 2: $\mathbf{n}_{\beta\sigma}\cdot D\nabla C_\beta = 0$ at $A_{\beta\sigma}$ (no diffusion across the solid interface)

I.C. $C_{i\beta}=C_{i\beta o}\left(\mathbf{x}\right)$ at $t=0$ and $\mathbf{x}\in\Omega$

where C_i is the concentration of component *i*, *D* is the molecular diffusion coefficient, and the subscript *eq* indicates equilibrium condition. The subscript β indicates that the equations are written for the moving phase.

Making the following major assumptions:

1. Constant temperature
2. β = aqueous phase; σ = solid phase; γ = non-aqueous phase; the σ, γ phases are assumed to be immobile: $\mathbf{u}_\sigma$ & $\mathbf{u}_\gamma = 0$ (**u** indicates velocity). This is equivalent to oil at residual saturation.
3. Incompressible aqueous phase and no-slip $\mathbf{u}_\beta = 0$ at $A_{\beta\gamma}$ and $A_{\beta\sigma}$ (A_{ij} represents the interface between phases *i* and *j*)
4. Fast kinetics such that equilibrium concentration is quickly established at the β-γ interface
5. No-slip boundary conditions ($\mathbf{u}$ = 0 at the β-σ and β-γ interfaces), and ignoring variations of molecular diffusivity within the averaging volume

The macroscopic quantities of interest such as concentrations and velocities, are expressed as a sum of their intrinsic average and a random fluctuation under the condition that they vary smoothly in space:

$$C_\beta=\left\langle C_\beta\right\rangle^\beta+C'_\beta \qquad \mathbf{u}_\beta=\left\langle \mathbf{u}_\beta\right\rangle^\beta+\mathbf{u}'_\beta$$

where $\left\langle C_\beta\right\rangle=\varepsilon_\beta\left\langle C_\beta\right\rangle^\beta$ **with** ε_β = volume fraction (saturation) of phase β. **The superscript β indicates that it is an intrinsic average (i.e. an average over only the volume of β phase in the averaging volume), as opposed to the superficial average $\left\langle C_\beta\right\rangle$, which is the average over the entire averaging volume.**

This yields the averaged equation

$$\underbrace{\varepsilon_\beta\frac{\partial\left\langle C_\beta\right\rangle^\beta}{\partial t}}_{\text{accum}}+\underbrace{\nabla\cdot\left\langle\mathbf{u}'_\beta C'_\beta\right\rangle}_{\text{dispersion}}+\underbrace{\varepsilon_\beta\left\langle\mathbf{u}_\beta\right\rangle^\beta\cdot\nabla\left\langle C_\beta\right\rangle^\beta}_{\text{convection}}=$$

$$\underbrace{\nabla\cdot\left[D\left(\varepsilon_\beta\nabla\left\langle C_\beta\right\rangle^\beta+\frac{1}{V}\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}C'_\beta dA+\frac{1}{V}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}C'_\beta dA\right)\right]}_{\text{diffusion}}$$

$$\underbrace{+\frac{1}{V}\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}\cdot D\nabla C'_\beta dA+\frac{1}{V}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot D\nabla C'_\beta dA+\frac{1}{V}\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}\cdot D\nabla\left\langle C_\beta\right\rangle^\beta dA+\frac{1}{V}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot D\nabla\left\langle C_\beta\right\rangle^\beta dA}_{\text{interfacial flux}}$$

And the deviation equation

$$\frac{\partial C'_\beta}{\partial t}-\varepsilon_\beta^{-1}\nabla\cdot\left\langle\mathbf{u}'_\beta C'_\beta\right\rangle+\mathbf{u}_\beta\cdot\nabla C'_\beta+\mathbf{u}'_\beta\cdot\nabla\left\langle C_\beta\right\rangle^\beta=\nabla\cdot\left(D\nabla C'_\beta\right)$$

$$-\varepsilon_\beta^{-1}\nabla\cdot\left[D\left(\frac{1}{V}\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}C'_\beta dA+\frac{1}{V}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}C'_\beta dA\right)\right]$$

$$-\varepsilon_\beta^{-1}\left(\begin{array}{l}\frac{1}{V}\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}\cdot D\nabla C'_\beta dA+\frac{1}{V}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot D\nabla C'_\beta dA\\+\frac{1}{V}\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}\cdot D\nabla\left\langle C_\beta\right\rangle^\beta dA+\frac{1}{V}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot D\nabla\left\langle C_\beta\right\rangle^\beta dA\end{array}\right)$$

Effective Mass Transfer Coefficient

The solution of averaged and deviation equations is achieved by constructing a closure problem, in which we impose a "closure condition" such that the summation of the average and deviation quantities satisfies a global mass balance conditions.

Substituting the closure condition $C_\beta{}'=-s_\beta\left(\left\langle C_\beta\right\rangle^\beta-C_{eq}\right)$ **into the deviation equation**

$$\left[\mathbf{u}_\beta-2D\frac{\nabla\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}{\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}\right]\cdot\nabla s_\beta=\left[\mathbf{u}_\beta\cdot\frac{\nabla\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}{\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}+D\frac{\nabla^2\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}{\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}\right]s_\beta-\mathbf{u}_\beta{}'\cdot\frac{\nabla\left\langle C_\beta\right\rangle^\beta}{\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}+D\nabla^2 s_\beta-\varepsilon_\beta{}^{-1}\alpha$$

Substituting into the averaged equation, we get:

$$\left\langle\mathbf{u}_\beta\right\rangle\cdot\nabla\left\langle C_\beta\right\rangle^\beta=-\nabla\cdot\left\langle\mathbf{u}_\beta{}'s_\beta\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)\right\rangle+D\varepsilon_\beta\nabla^2\left\langle C_\beta\right\rangle^\beta+\alpha\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)$$

Defining:

$$\alpha=\frac{D}{V\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}\nabla\cdot\left[\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}s_\beta\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)dA+\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}s_\beta\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)dA\right]$$

$$+\frac{D}{V\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)}\left[\int_{A_{\beta\gamma}}\mathbf{n}_{\beta\gamma}\cdot\nabla\left[s_\beta\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)\right]dA+\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot\nabla\left[s_\beta\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)\right]dA\right]$$

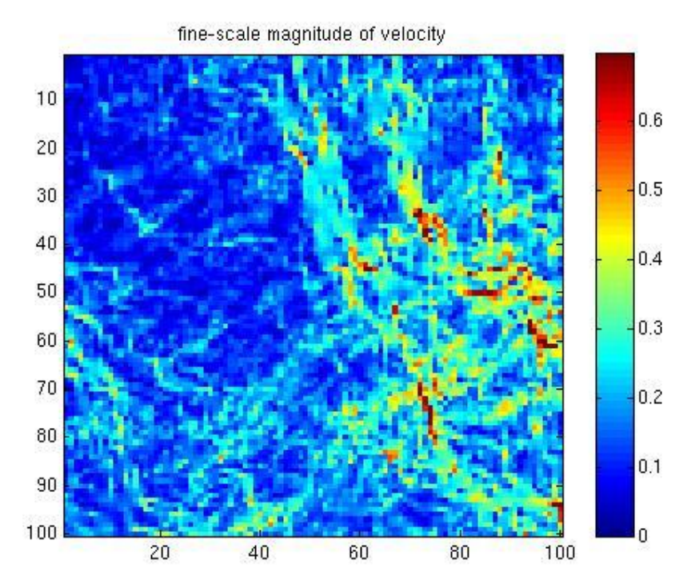
α = effective interfacial mass transfer coefficient

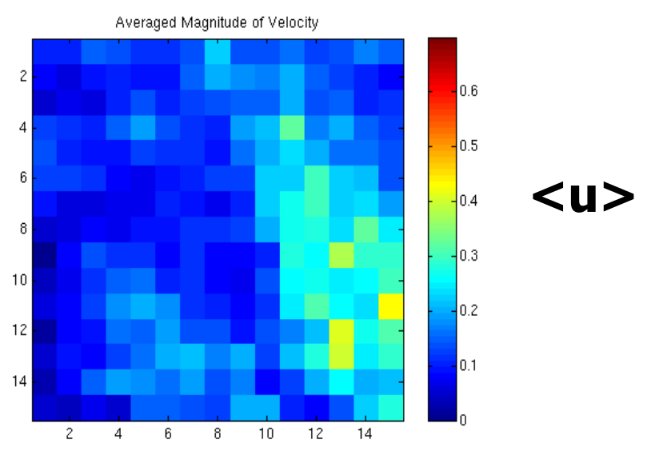
Once a is calculated, calculate effective mass transfer coefficient

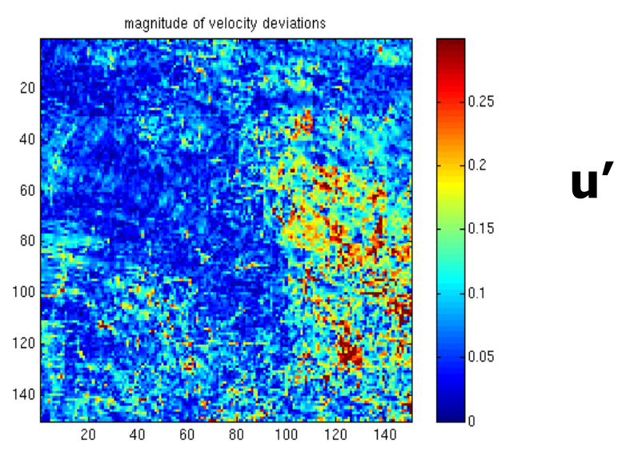
$$\left\langle\mathbf{u}_\beta\right\rangle\cdot\nabla\left\langle C_\beta\right\rangle^\beta=K_{eff}\left(C_{\max}-\left\langle C_\beta\right\rangle^\beta\right)$$

Approach

STEP #1: Modeling the velocity field







obtain from fine-scale simulations

STEP #2: Volume-Averaging

- **Couple Eqs.1 & 2 (solve for <C>, s_β, and α)**

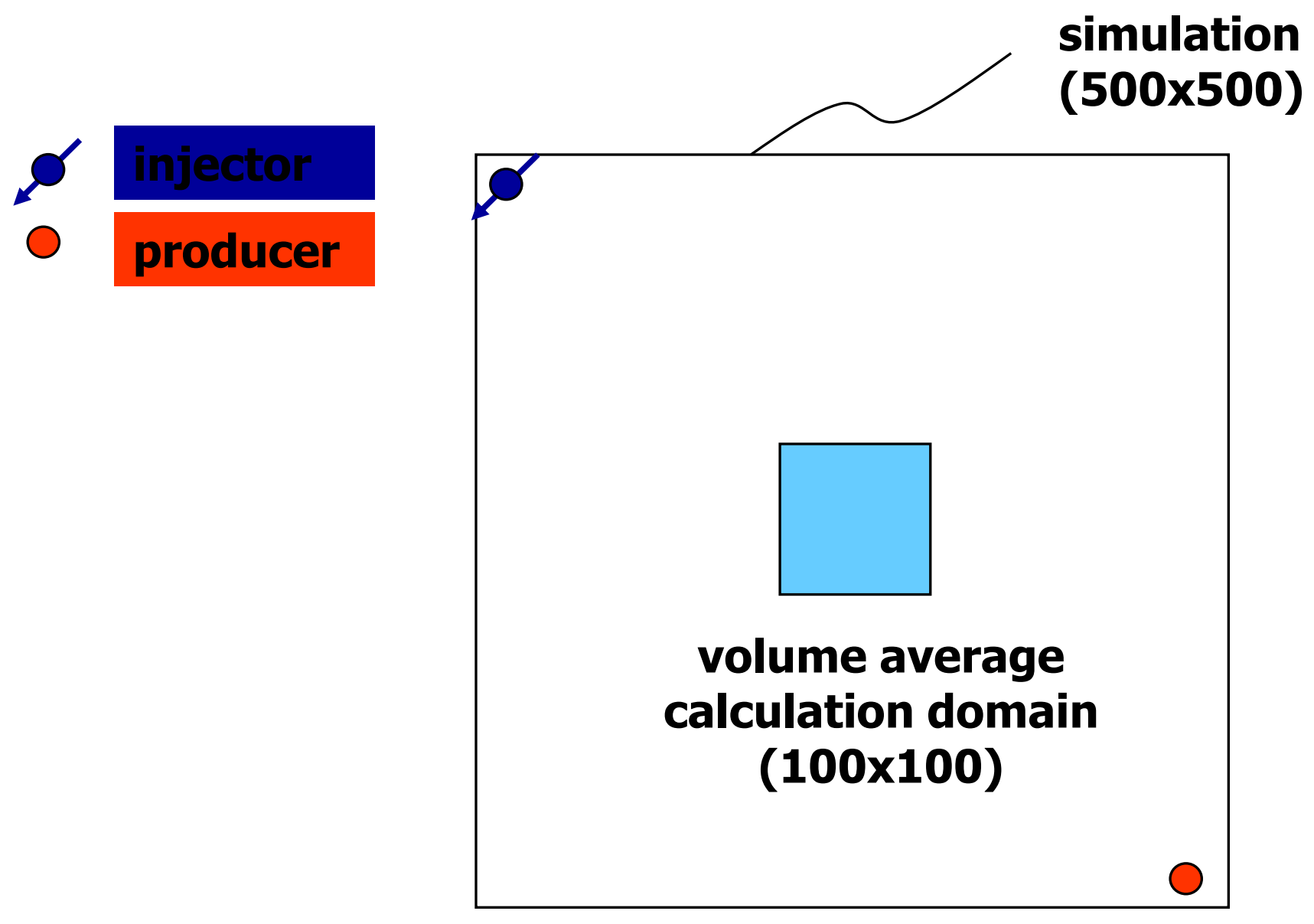
Optimization Procedure

- **Initialize $<C_\beta>^\beta$ = arithmetic average**
- **Compute** $s_\beta{}^{old}=\left(C_\beta-\left\langle C_\beta\right\rangle^\beta\right)/\left(C_{eq}-\left\langle C_\beta\right\rangle^\beta\right)$
- **Compute α**
- **Compute error ε** $\varepsilon=D\nabla^2 s_\beta+bs_\beta-\mathbf{a}\cdot\nabla s_\beta-c-e\alpha$
- **Update $<C_\beta>^\beta$** $\left[\left\langle C_\beta\right\rangle^\beta\right]^{new}=\left[\left\langle C_\beta\right\rangle^\beta\right]^{old}+perturbation$
- **Accept or reject the new guess of $<C_\beta>^\beta$**

STEP #3: Evaluate K_{eff} $\left\langle\mathbf{u}_\beta\right\rangle\cdot\nabla\left\langle C_\beta\right\rangle^\beta=K_{eff}\left(C_{\max}-\left\langle C_\beta\right\rangle^\beta\right)$

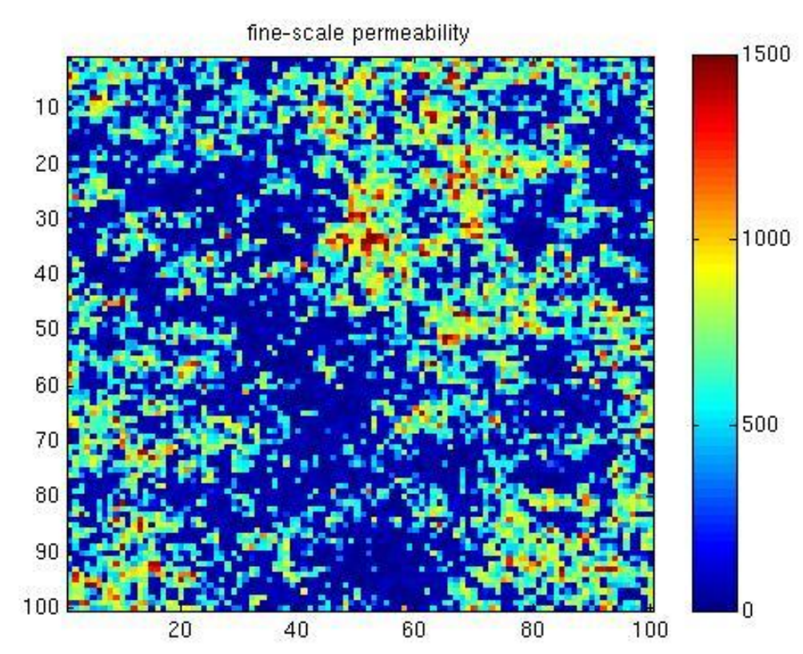
Results

Model Setup



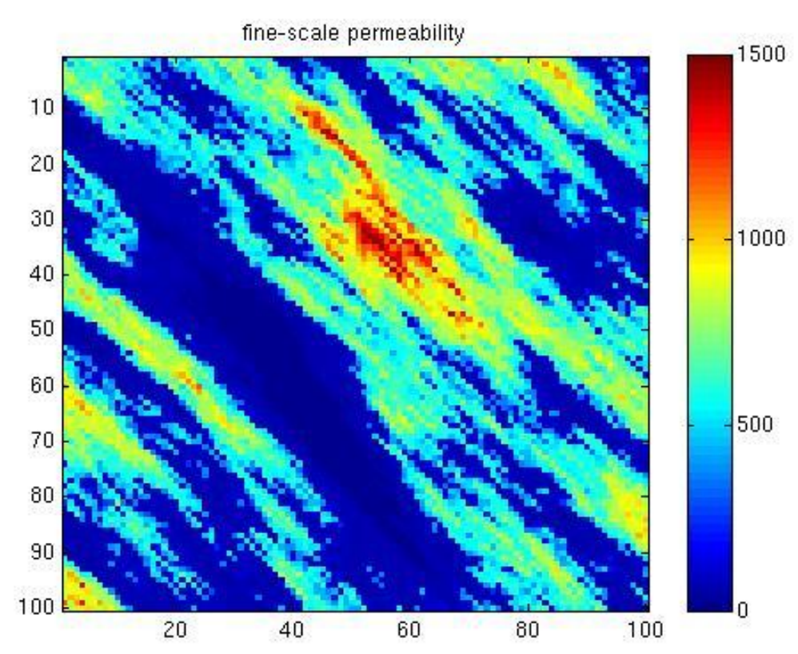
Two Cases

Case 1: short correlation length



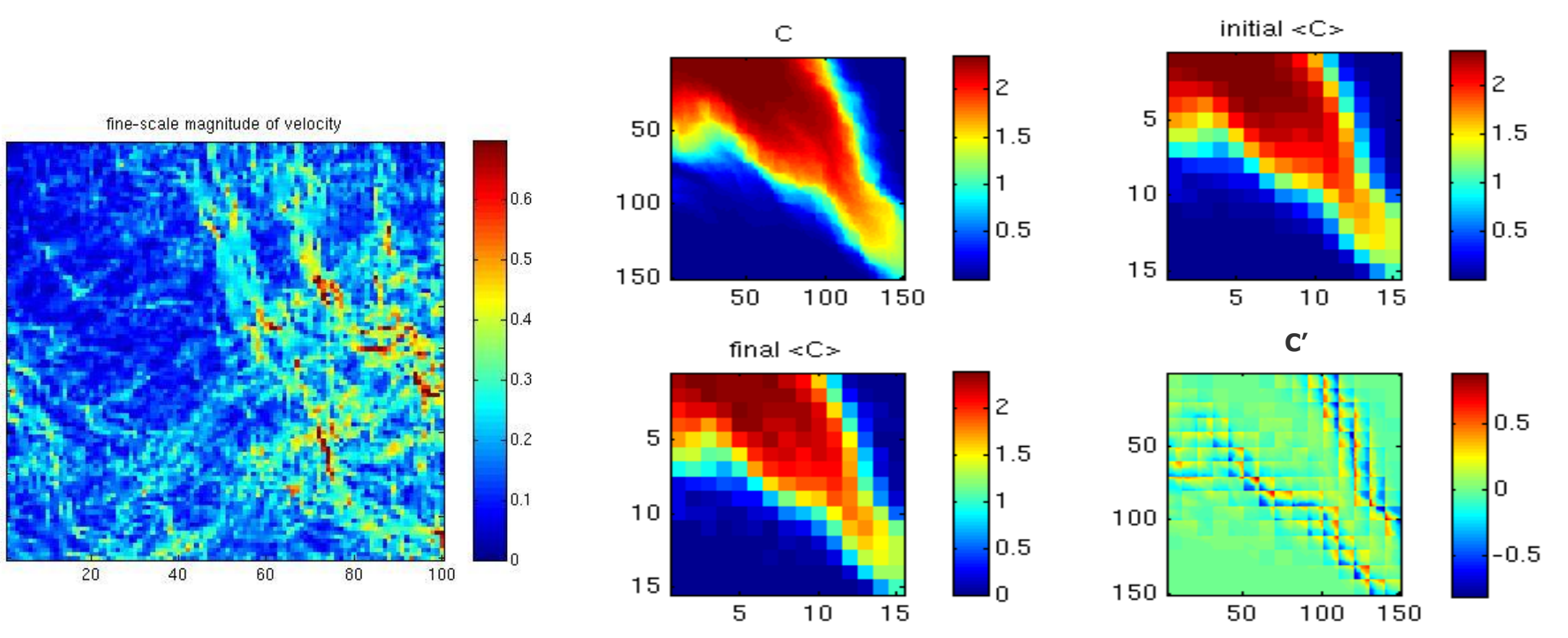
range = 2 units (isotropic)

Case 2: long correlation length

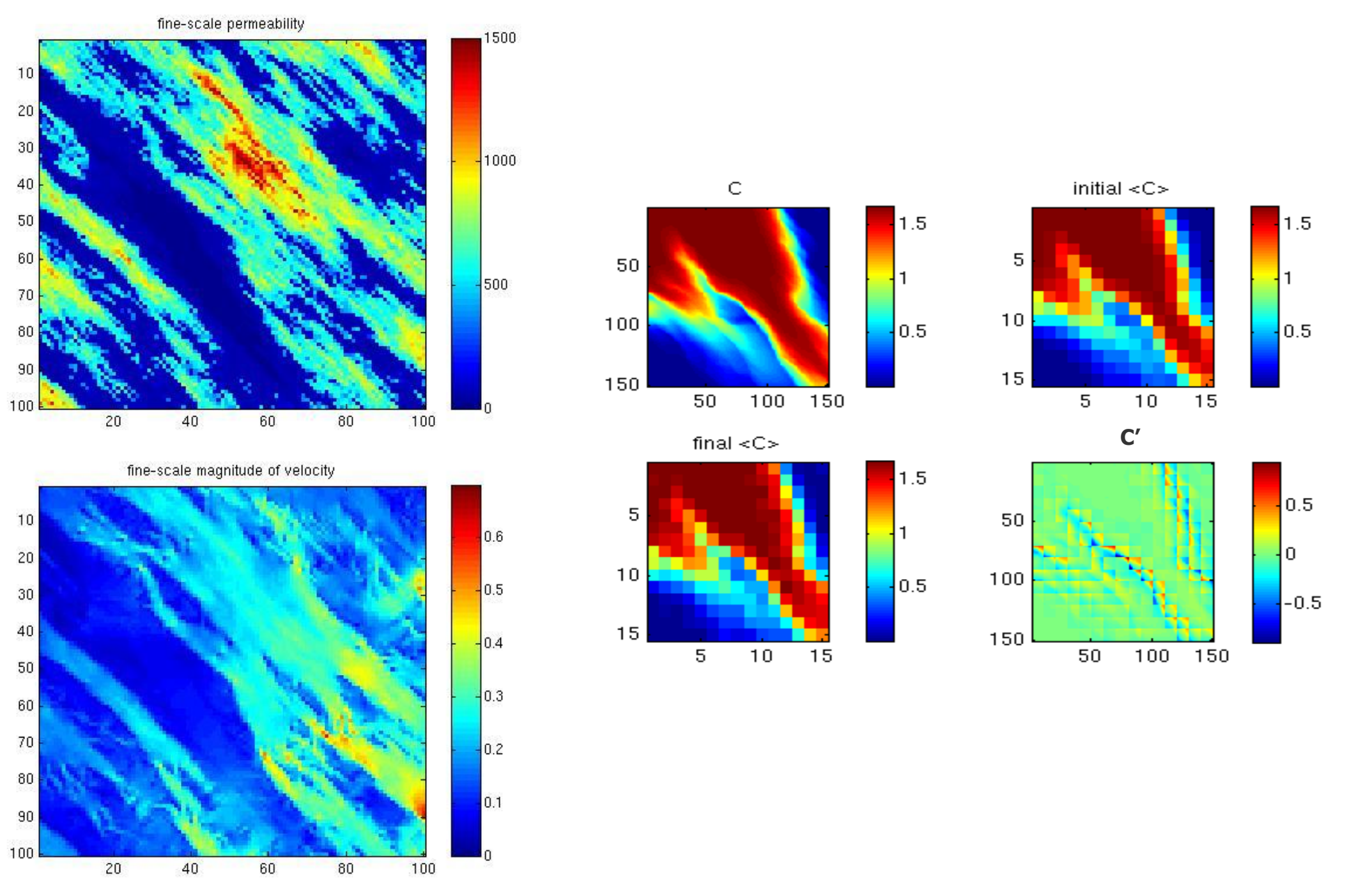


range = 50 units (max)
= 10 units (min)

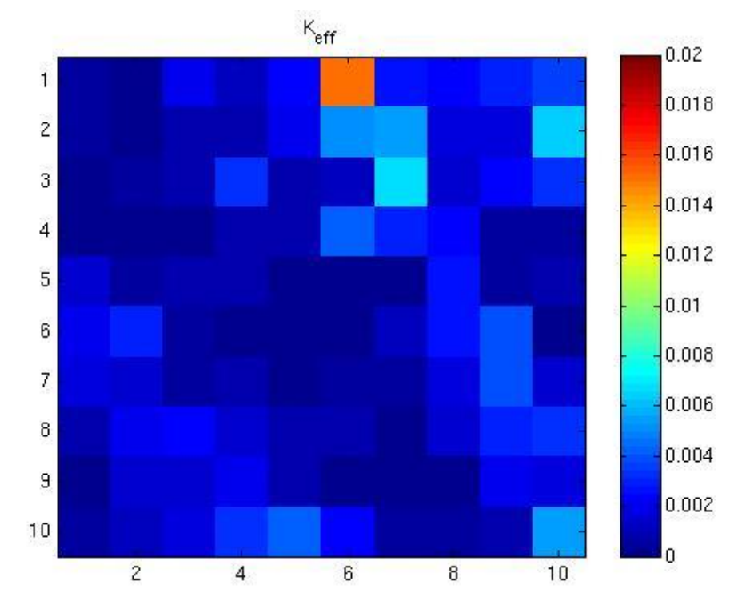
Result for short correlation length



Result for long correlation length

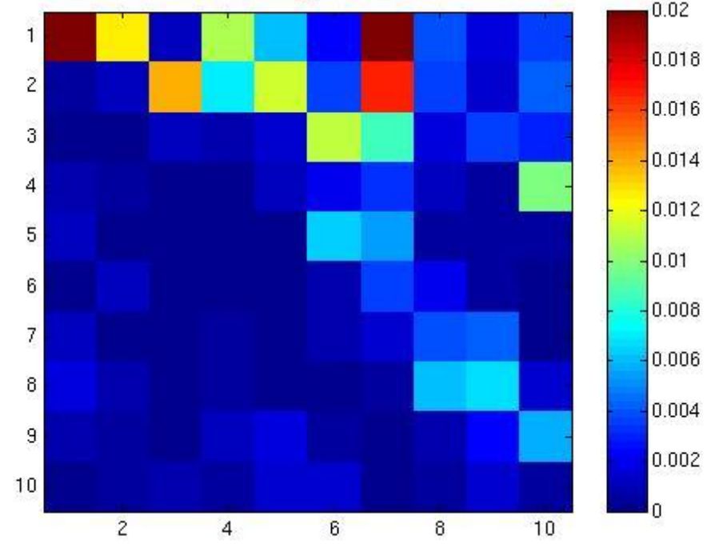


Effective Mass Transfer Coefficient Keff



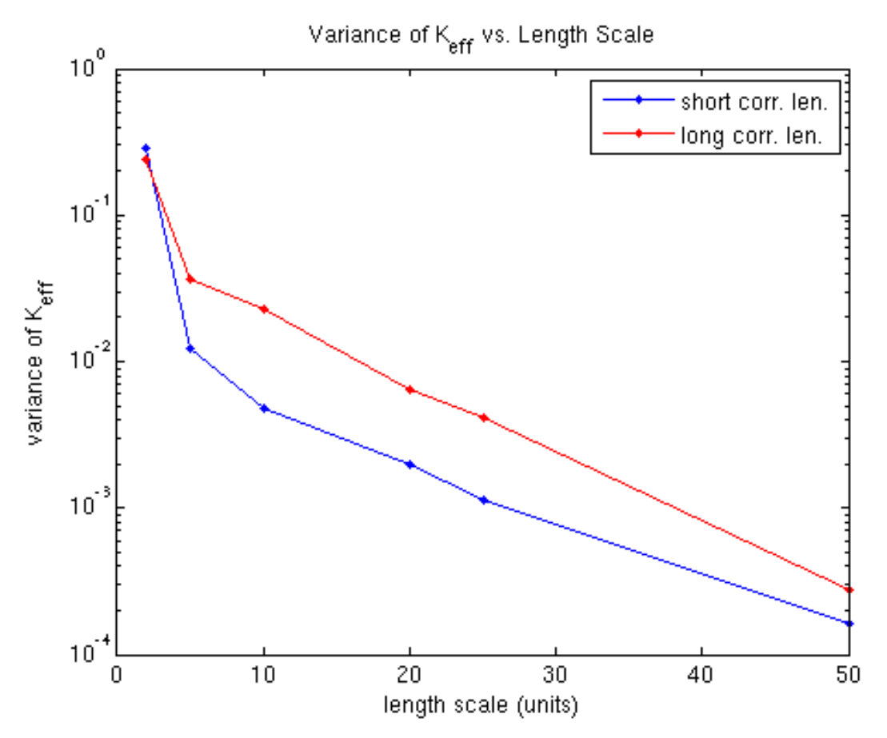
Case 1: Small Correlation Lengths

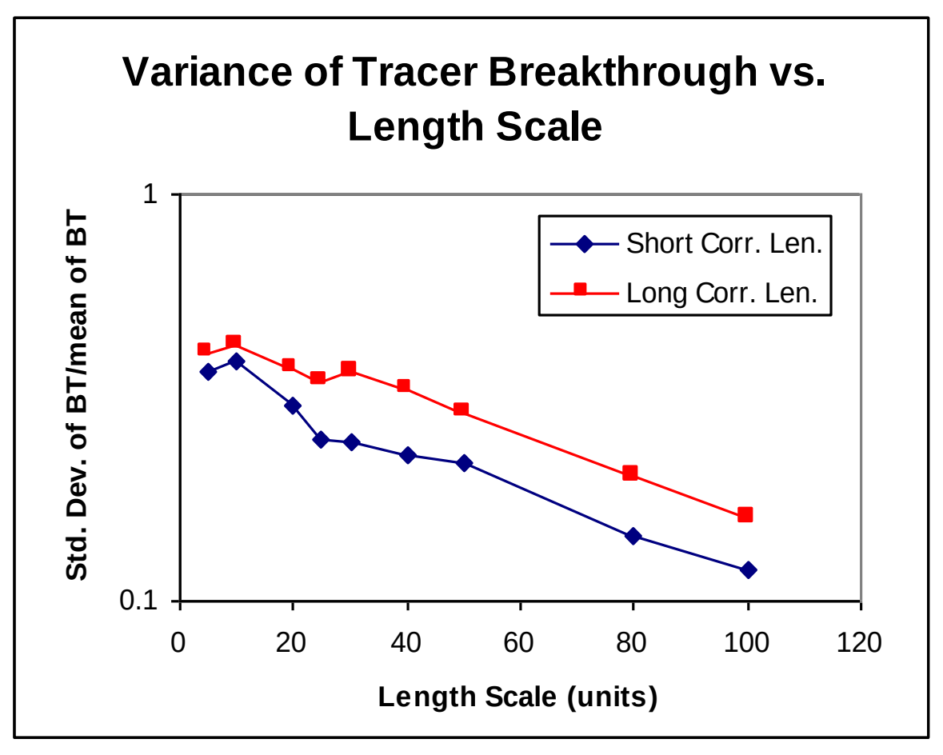
More variability



Case 2: Large Correlation Lengths

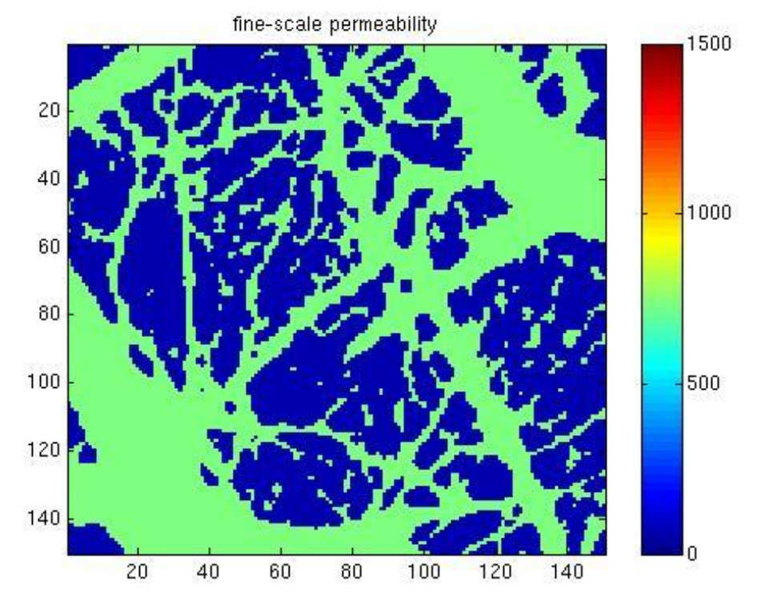
Scaling of Keff



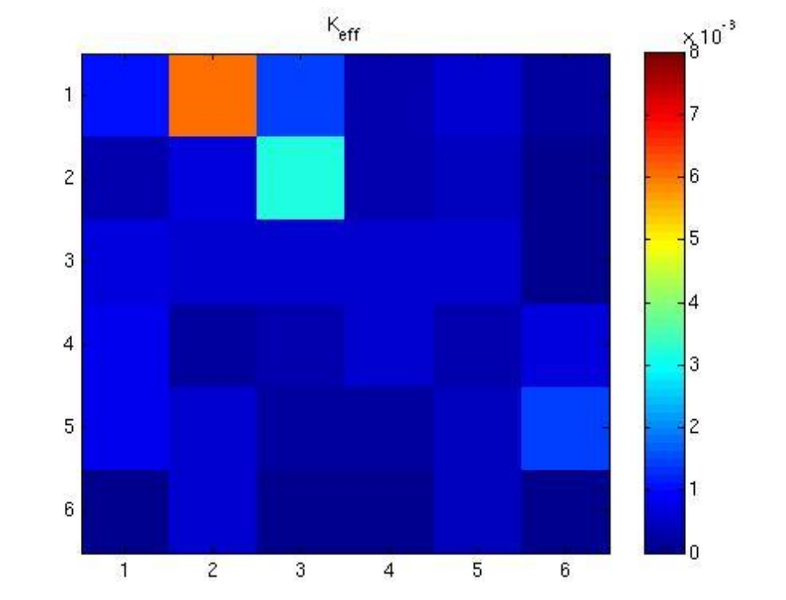


Scaling of Keff ~ Scaling of tracer recovery

Another example – Fractal media



Hurst exponent (H) = 0.325 by box-counting

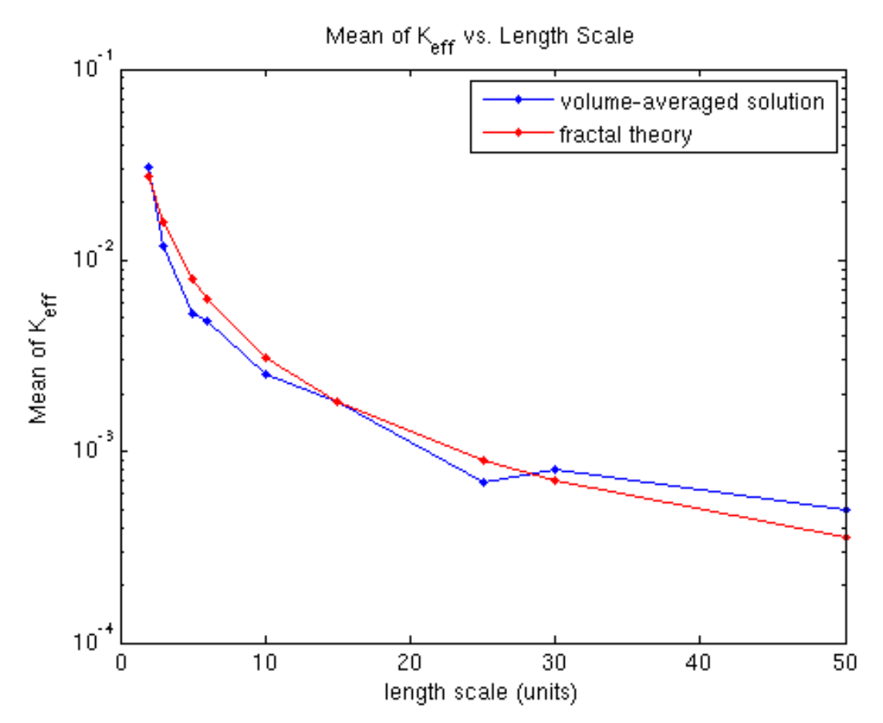


Map of K_{eff}

- **Transport mechanisms accounted for in K_{eff}**
  - **Diffusion (minor)**
  - **Dispersion (major)**
- **Fractal theory for dispersivity (Sahimi, 1993)**

$$\alpha_L=DL=t^{2H-1}\approx L^{2H-1}$$
$$\therefore D=L^{2H-2}$$

α_L = dispersivity
D = dispersion coefficient
L = length scale
t = time scale
H = Hurst exponent



Scaling of Keff ~ Scaling of D

References

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Acknowledgements

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